Differentiated Rate Sceduling for MIMO Broadcast Channels*

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Abstract

We consider the problem of differentiated rate scheduling for the fading MIMO Gaussian broadcast channel, in the sense that the rates required by different users must satisfy certain rational rate constraints. When full channel state information (CSI) is available at the transmitter, the problem can be readily solved using dirty paper coding (DPC) and convex optimization techniques on the dual multiple-access channel (MAC). However, since in many practical applications full CSI is not feasible, and since the computational complexity may be prohibitive when the number of users is large, we focus on two simple schemes that require very little CSI: time-division opportunistic (TO) beamforming where in different time-slots the transmitter performs opportunistic beamforing only to users requiring the same rate, and weighted opportunistic (WO) beamforing where the random beams are assigned to those users having the largest *weighted* SINR. In both cases we determine explicit schedules to guarantee the rate constraints and show that, in the limit of a large number of users, the throughput loss compared to the unconstrained sum-rate capacity tends to zero. As a side result, we show that, in this regime, the sum-rate of opportunistic beamforming converges to the optimal sum-rate achieved by DPC, which is a stronger result than the order-optimal results of [10, 13].

1 Introduction

The down-link scheduling in cellular systems is known to be one major bottleneck for future broadband communication systems. From an information-theoretic perspective, broadcast channels [1], and in particular the Gaussian broadcast channel, can be used to model the down-link in a cellular system. There exist an abundance of information-theoretic results describing the limits of the achievable rates to the users in single-input single-output (SISO) Gaussian broadcast channels (see e.g., [2, 3]). For example in a homogeneous network, i.e., a network where the fading and noise distributions of all the users are identical, if the transmitter wants to maximize the throughput (or the sum of the rates to all the receivers), it is well known that the optimal strategy is to transmit to the user with the best channel condition at each channel use. This is often referred to as the *opportunistic* transmission strategy [4].

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More recently, there has been growing interest in the use of multiple antennas (at the transmitter, receiver, or both) for wireless communication systems. The initial focus has been on point-to-point communications where it has been shown that the use of multiple antennas can significantly increase the rate and reliability of a wireless communication link. Given this, both the research and industrial communities have begun to study the use of multiple antenna systems in wireless networks. A most obvious application is in cellular systems where the use of multiple antennas at the base station can potentially increase the capacity of each cell. This has led to an interest in the multiple-input multiple-output (MIMO) Gaussian broadcast channel, where the transmitter and the various users may be equipped with multiple transmit and receive antennas, respetively. First, the sum-rate of the MIMO broadcast channel, i.e., the maximum possible sum of the rates to all users [5], and then the entire capacity region [6] were shown to be achieved by an interference cancellation scheme referred to as *dirty paper coding* [7].

Thus, from a theoretical point of view, the limits of reliable communication in MIMO Gaussian broadcast channels is well understood. Fortunately, the same is true if one takes a *computational* point of view. In other words, it is well known how to computationally obtain any point on the boundary of the capacity region. In a nutshell, the methodology can be explained as follows. Each point on the boundary of the capacity region is characterized by a set of covariance matrices (corresponding to how DPC is used at that particular boundary point). To obtain the desired covariance matrices one may construct a *dual* multiple-access (MAC) system [8] where, due to the polymatroid structure of the problem, the solution can be found via standard convex optimization techniques [9].

In view of the aforementioned results, it would appear that there is not much left to do for MIMO broadcast channels. However, a crucial assumption in all these results is that the channel coefficients to all the users be known–an assumption referred to as full channel state information (CSI)–at the transmitter. In fact, it is easy to show that with no CSI at the transmitter there is no capacity gain to be had by employing multiple antennas at the transmitter (provided all the users have single antennas) [10]. However, in practice, obtaining full CSI at the transmitter may not be feasible, especially for systems where the number of users is large and/or the users are mobile so that the channel coefficients vary rapidly with time.

Furthermore, when the number of users is large, the computational complexity of DPC, and even the convex optimization steps required to determine the optimal covariance matrices from the dual MAC, may become prohibitively large. Therefore there is interest in developing *simple* schemes that require *little* CSI at the transmitter, yet deliver on most of the capacity offered by the MIMO broadcast channel. One such scheme that achieves most of the MIMO broadcast capacity in certain regimes is described in [10].

In homogenous networks, the sum-rate point is a symmetrical point on the boundary of the capacity region and so treats all the users equally. In systems which are provisioned to provide differentiated services to different users, the transmitter has to give different services (or rates) to different subsets of receivers, and yet at the same time, maximize the throughput (see e.g., [11] for a discussion of the SISO case). Giving differentiated rates to users clearly means operating at non-symmetrical boundary points of the capacity region. As mentioned earlier, this problem can, in principle, be solved since the duality to the MAC allows one to attain any point on the capacity region.

However, since this solution requires full CSI at the transmitter and potentially prohibitive computations when the number of users is large, the main goal of this paper is to develop *simple* schemes, that require very *little* CSI, give *differentiated rates* to the users, and that operate *close* to the boundary of the capacity region. We will also be interested in quantifying the rate loss, compared to the sum rate, for various differentiated rate schemes. In this sense, the current paper can be considered as the MIMO extension of our earlier work [12].

We should also mention that in this paper we will only be dealing with homogenous networks, in the sense that the SNR of the different users are assumed to have the same probability distribution. Of course many networks are, in fact, heterogenous, with different users having different distributions for their SNRs. The methodology of this paper (and many of the results, we suspect) can be straightforwardly carried over to the heterogenous case, with the caveat that the development will be much more involved and cumbersome. For this reason, and for reasons of space, although quite important in practice, we deem the heterogenous case beyond the scope of the current paper. We only remark that a common practice to make a heterogenous network appear homogenous is to use an appropriate power control (after which all our results will directly apply).

The remainder of the paper is organized as follows. Section 2 describes the model we are considering and obtains various scaling laws on the optimal sum-rate achieved by dirty paper coding, as well as the sum-rate achieved by opportunistic beamforming. Section 3 states the main problem and gives the main results of the paper. A simple simulation is presented in Section 4 and the paper concludes with Section 5.

2 The Model

Consider a fading Gaussian broadcast channel with M antennas at the transmitter and n users, each with $N = 1$ receive antennas.[1] The channels to each user are assumed to be block fading with a coherence interval of T; in other words, the channels remain constant for T channel uses after which they change to independent values.[2] Furthermore, over different users the fading is assumed to be independent

Thus, during any coherence interval, the signal to the i-th user, $i = 1, 2, \ldots, n$, can be written as

$$x_i(t) = \sqrt{P} H_i s(t) + w_i(t), \quad t = 1, \ldots, T \tag{1}$$

where $H_i \in \mathcal{C}^{1\times M}$ is constant during the coherence interval and has iid $\mathcal{CN}(0,1)$ entries, $w_i(t)$ is additive white noise with distribution $\mathcal{CN}(0,1)$, $s(t) \in \mathcal{C}^{M\times 1}$ is the transmit symbol satisfying $E\|s(t)\|^2 = 1$ and P is the total transmit power.

2.1 Throughput Scaling Laws

In point-to-point multi-antenna systems the throughput scaling is often equivalent to the "multiplexing gain" defined as $\lim_{\text{SNR}\to\infty} \frac{C}{\log \text{SNR}}$. However, in broadcast channels two different throughput scaling laws can be envisioned.

Theorem 1 (Large Power Regime). *Consider the fading MIMO Gaussian broadcast channel of Section 2. Then for fixed M and n, we have*

$$\lim_{P\to\infty} \frac{C_{sum}}{\log P} = M, \tag{2}$$

where C_{sum} refers to the maximum possible sum of the rates to all n users.

[1]It is possible to extend our results to $N \neq 1$ in a straightforward fashion. However, for simplicity, we shall not do so here. From a practical point of view $N = 1$ is also very reasonable.

[2]We should remark that, although the assumption of a constant channel for T channel uses is critical, the requirement that the channels vary independently from one coherence interval to the next is not.

Theorem 2 (Large Number of Users Regime). *[13] Consider the fading MIMO Gaussian broadcast channel of Section 2. Then for fixed M and P, we have*

$$\lim_{n\to\infty} \frac{C_{sum}}{\log\log n} = M, \tag{3}$$

where C_{sum} refers to the maximum possible sum of the rates to all n users.

These are clearly two very different regimes. We argue that, from a practical perspective, the latter regime may be more interesting. There are three reasons that come to mind.

1. Many practical systems operate with a large number of per-cell users (n could be in the hundreds, whereas M may be no more than two, three or four).
2. Significant rates can be obtained even at low to moderate transmit powers P.
3. The first gain requires channel knowledge with very high fidelity at the transmitter (indeed a fidelity that grows with the transmit power) [14], whereas the latter requires very little CSI (see, e.g., [10]).

In view of the above, in this paper we will focus on the *large* n regime.

2.2 Opportunistic Beamforming

In this section we briefly describe a simple scheme that achieves most of the broadcast sum-rate in the large n regime, yet requires very little CSI at the transmitter. The idea is based on transmitting M random beams and is described in [10] (a similar construction, albeit with little analysis, appears in the appendix of [15]).

Basically, during any coherence interval the transmitter chooses M random orthonormal vectors $\phi_m \in \mathcal{C}^{M\times 1}$ according to an isotropic distribution and then transmits the vector

$$s(t) = \sum_{m=1}^{M} \phi_m s_m(t), \tag{4}$$

where each $s_m(t)$ is a scalar signal intended for one of the users. Assuming the users know their own channel coefficients (a much more reasonable assumption than the transmitter knowing *all* the channel gains to the different users), each user can compute its signal-to-interference-plus-noise-ratio (SINR) for every beam as

$$\text{SINR}_{m,i} = \frac{|H_i\phi_m|^2}{\frac{M}{P} + \sum_{l\neq m} |H_i\phi_l|^2}. \tag{5}$$

If each user (or, in fact, only those users who have favorable SINRs) feeds back its best SINR and corresponding beam index to the transmitter, the transmitter can assign each beam to the user that has the best SINR for that beam. (This is the jist of the idea—for more details see [10].)

For this scheme the following result can be shown.

Theorem 3. *[10] Consider the fading MIMO Gaussian broadcast channel of Section 2 and let C_{ob} denote the sum-rate obtained by the opportunistic beamforming technique described above. Then for fixed P and M*

$$\lim_{n\to\infty} \frac{C_{ob}}{\log\log n} = M. \tag{6}$$

However, for fixed n and M

$$\lim_{P\to\infty}\frac{C_{ob}}{\log P}=0. \tag{7}$$

In other words, opportunistic beamforing is order-optimal in the large n regime, but not in the large P regime. The reason is that opportunistic beamforming is interference dominated and so the sum-rate does not scale with the logarithm of the power. (In fact, to obtain the multiplexing gain of M at high power requires essentially eliminating the interference, such as is done by a zero-forcing solution.)

2.3 Tighter Scaling Laws

In fact, one can give a tighter result than the growth rates described so far.

Theorem 4. *Consider the fading MIMO Gaussian broadcast channel of Section 2. For fixed P and M*

$$C_{sum}=M\log\log n+M\log\frac{P}{M}+o(1), \tag{8}$$

where C_{sum} refers to the maximum possible sum of the rates to all n users and $o(1)$ is with respect to growing n.

Proof: We can give a quick sketch of the proof. Using the duality with the MAC one can write

$$\begin{aligned}
C_{sum} &= E\max_{\sum_{i=1}^n P_i=P,P_i\geq 0}\log\det\left(I_M+\sum_{i=1}^N H_i^*P_iH_i\right)\\
&\leq E\max_{\sum_{i=1}^n P_i=P,P_i\geq 0}\log\det\left(I_M+\left(\max_i\|H_i\|^2\right)\underbrace{\sum_{i=1}^N\phi_i^*P_i\phi_i}_{=X}\right), \quad \phi_i=\frac{H_i}{\|H_i\|}\\
&\leq E\max_{\text{trace}(X)=P,X\geq 0}\log\det\left(I_M+\left(\max_i\|H_i\|^2\right)X\right),\\
&= E\, M\log\left(1+\frac{P}{M}\left(\max_i\|H_i\|^2\right)\right)
\end{aligned}$$

It can be shown that the random variable $\max_i\|H_i\|^2$ with high probability behaves as $\log n$. A careful analysis of the expectation in the last equation shows that (see [16] for the details)

$$C_{sum}\leq M\log\log n+M\log\frac{P}{M}+o(1).$$

To complete the proof we need a lower bound that has the same behavior. It turns out that the desired lower bound can be obtained by employing opportunistic beamforming. The required result is the next theorem. ■

Theorem 5. *Consider the setting of Theorem 4. Then if we use opportunistic beamforming*

$$C_{ob}=M\log\log n+M\log\frac{P}{M}+o(1). \tag{9}$$

Proof: We sketch the proof and refer to [16] for the details. Following [10] we can write

$$\begin{aligned} C_{sum} &= ME\log\left(1+\frac{P}{M}\max_i \text{SINR}_{1,i}\right) \\ &\geq M\log\left(1+\frac{P}{M}(\log n - 2M\log\log n)\right)\underbrace{\text{Prob}\left(\max_i \text{SINR}_{1,i} > \log n - 2M\log\log n\right)}_{=1-O(\frac{1}{\log n})} \\ &= M\log\log n + M\log\frac{P}{M} + o(1). \end{aligned}$$

■

Theorems 4 and 5 imply

$$\lim_{n\to\infty}(C_{sum} - C_{ob}) = 0, \tag{10}$$

which is a much stronger result than being simply order optimal.

3 Problem Statement and Solution

We are interested in giving different rates to the different users. Thus, assume that the n users are divided into K groups, each with $\alpha_k n$ users ($\sum_{k=1}^K \alpha_k = 1$) and each require a different rate. In particular, $R^k/R^K = \beta_k$, $k = 1,\ldots,K-1$, where R^k is the rate required for group k and β_k represents the rational rate requirements. We are now in a position to state our main problem.

Problem 1. *Consider the fading MIMO Gaussian broadcast channel of Section 2. Let R_i denote the rate to i-th user and R^k denote the rate to all users in group k. Then construct a transmission scheme such that*

$$\max \sum_{i=1}^n R_i$$
$$\textit{subject to } \frac{R^k}{R^K} = \beta_k, \quad k = 1,\ldots,K-1$$

Clearly, the solution to Problem 1 is given by the intersection of the line $R^k/R^K = \beta_k$, $k = 1,\ldots,K-1$ with the boundary of the capacity region of the broadcast channel. It can numerically be solved using bisection in the following way.

Algorithm 1.

1. *Choose a set of rates R'^k satisfying the rate constraints*
2. *By appealing to the dual MAC, solve the problem.* $\min\sum p_i$, *subject to the rates R'^k.*
3. *If the minimum sum of powers* $\min\sum p_i$ *is less than P, then the rate vector is achievable. Increase the rate proportionately and go to 2.*
4. *Otherwise decrease the rates proportionately and go to 2.*

While this is all fine, the algorithm is computationally-intensive (even though the problem in step 2 is convex, it is time-consuming if n is very large), requires full CSI and still requires DPC. Thus, in the remainder of the paper we will focus on simple schemes.

3.1 Time-Division Opportunistic (TO) Beamforming

Assume we divide each coherence interval into K slots of duration t_k each, $k = 1, \ldots, K$. During the k-th subinterval the transmitter performs opportunistic beamforming to *only* the $\alpha_k n$ users in the k-th group. It is not hard to convince oneself that to satisfy the rational rate constraints, we must have

$$\frac{t_k}{T} = \frac{\alpha_k \beta_k}{\sum_{l=1}^{K} \alpha_l \beta_l}, \quad k = 1, \ldots, K \tag{11}$$

We now have the following result.

Theorem 6. *Consider the fading MIMO Gaussian broadcast channel of Section 2. Let M, P, and the α_k and β_k's be fixed and let the subintervals be chosen as (11). Then the rational rate constraints are met and*

$$\lim_{n \to \infty} (C_{sum} - C_{tdob}) = 0, \tag{12}$$

where C_{tdob} represents the sum-rate for the time-division opportunistic scheme.

Proof: We provide only the sketch. For details see [16]. That the rational rate constraints are met is fairly obvious. As for C_{tdob}:

$$\begin{aligned} C_{tdob} &= \sum_{k=1}^{K} \frac{\alpha_k \beta_k}{\sum_{l=1}^{K} \alpha_l \beta_l} \left(M \log\log n\alpha_k + M \log \frac{P}{M} + o(1) \right) \\ &= \sum_{k=1}^{K} \frac{\alpha_k \beta_k}{\sum_{l=1}^{K} \alpha_l \beta_l} \left(M \log\log n + \underbrace{\log\left(1 + \frac{\alpha_k}{\log n}\right)}_{=o(1)} + M \log \frac{P}{M} + o(1) \right) \\ &= M \log\log n + M \log \frac{P}{M} + o(1) \end{aligned}$$

■

Thus, asymptotically in n, TO beam-forming achieves the unconstrained sum-rate capacity while also satisfying the rational rate constraints.

3.2 Weighted Opportunistic (WO) Beamforming

Here we weigh the SINR of each user according to its group by a weight μ_k, $k = 1, \ldots, K$. Then during each coherence interval, the transmitter assigns the M random beams to the M users that have the largest *weighted* SINR.

In the WO beamforming scheme there are two questions to be answered. First, how to determine the weights such that the rational rate constraints are met. Here, unlike the TO case, the answer is not trivial. And second, what is the rate loss compared to the unconstrained sum-rate capacity of the broadcast channel itself.

Theorem 7. *Consider the fading MIMO Gaussian broadcast channel of Section 2. Consider the WO beamforming scheme with*

$$\mu_k = 1 + \frac{\log \beta_k}{\log n + (M-1)(1 - \log\log n)}. \tag{13}$$

Assuming, M, P, α_k's and β_k's are fixed, we have

$$\lim_{n\to\infty} \frac{R^k}{R^K} = \beta_k, \quad k = 1, \ldots, K. \tag{14}$$

Proof: Again we provide only the briefest sketch. For details see [16]. It can be shown that if a user in group k has the maximum weighted SINR, then its distribution is

$$p_k(x) = \frac{Me^{-x}(x + \frac{P}{M} + M - 1)}{P(1 + \frac{Mx}{P})^M} \left(1 - \frac{e^{-x}}{\left(1 + \frac{Mx}{\mathrm{P}}\right)^{M-1}}\right)^{\alpha_k n - 1} \prod_{l \neq k}^{\mathrm{K}} \left(1 - \frac{e^{-\frac{\mu_k}{\mu_l}x}}{\left(1 + \frac{\mu_k}{\mu_l}\frac{Mx}{\mathrm{P}}\right)^{M-1}}\right)^{\alpha_l n}. \tag{15}$$

The rate to a user in the k-th group is $M\int_0^\infty \log(1 + \frac{P}{M}x)p_k(x)dx$. Dividing this integral into three regions, namely between zero and $\log n - (M+3)\log\log n$, from $\log n + (M+3)\log\log n$ to infinity and the region between these two bounds, some careful calculation shows that

$$R^k = \frac{M\log\log n + M\log(\frac{P}{M})}{n\sum_{l=1}^K \alpha_l e^{(\frac{\mu_k}{\mu_l} - 1)(\log n + (M-1)(1 - \log\log n))}} + O\left(\frac{\log\log n}{n\log n}\right). \tag{16}$$

Using this expression we can easily prove (14). ■

The final result shows that, as in the case of TO beamforming, WO beamforming achieves the sum-rate of the unconstrained broadcast channel as $n \to \infty$.

Theorem 8. *Consider the setting of Theorem 7 and let C_{wob} denote the sum of the rates obtained by the weighted opportunistic beamforming scheme. Then.*

$$\lim_{n\to\infty} (C_{sum} - C_{wob}) = 0. \tag{17}$$

Proof: The idea of the proof is similar to that of Theorem 7. In fact, it can be shown that,

$$(C_{sum} - C_{wob}) = O\left(\frac{(\log\log n)^2}{\log n}\right). \tag{18}$$

For details see [16]. ■

4 Simulation Results

To gain some insight into the performance of the schemes described, we present a simple simulation result in this section. We consider the case of $K = 2$ users and require that $\frac{R^1}{R^2} = \beta_1 = 2$. Finally, we assume $M = 2$ transmit antennas at the base station, $\frac{P}{M} = 1$ (so that the system operates at 0 db) and vary the number of users from $n = 50$ to $n = 5000$. Figure 1(a) shows the ratio of the rates of two users in the two different groups when WO beamforming is used with μ_1 and μ_2 as in Theorem 7. As n increases the ratio converges to the desired value. In Figure 1(b) the sum rate of the WO and TO schemes are plotted and compared to that of the unconstrained opportunistic scheme in which the users are not divided into groups. For reference, we also plot $M\log\log n + M\log\frac{P}{M} = M\log\log n$ (since $\frac{P}{M} = 1$). The throughputs all converge to $M\log\log n + M\log\frac{P}{M}$, though the convergence rate is quite slow. Finally, we note that the WO scheme clearly outperforms the TO scheme, and has negligible performance loss compared to the unconstrained opportunistic scheme. (Similar conclusions for the SISO case were observed in [12].)

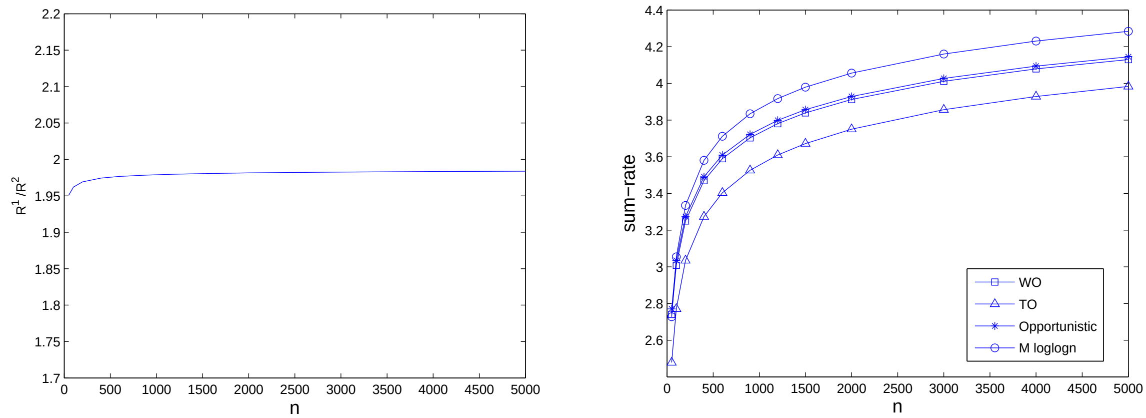

Figure 1: (a) ratio of the rates and (b) the sum-rate of WO and TO schemes for $M = 2$ and $\beta = 2$

5 Conclusion

We considered a homogenous fading MIMO Gaussian broadcast channel with n users demanding different rates. In our model we assumed users are divided into K groups each one of which demands a different rate and where the ratios of the rates of the groups are given. Users in each group have the same rate requirement. We considered the problem of scheduling to users to maximize the throughput of the system while maintaining the rate constraints. While the problem in its full generality can be solved it requires full CSI at the transmitter and high computational complexity. We therefore focused on two simple schemes that require very little CSI, namely, time-division opportunistic (TO) and weighted opportunistic (WO) beamforing. We gave explicit scheduling to guarantee the rate constraints. We further showed that the throughput loss due to these constraints tends to zero for both schemes as the number of users increases, with the performance of WO being significantly superior. Generalizations include considering a heterogenous network where the users are not statistically identical and where they may have more than $N = 1$ receive antennas.

References

[1] T. Cover, “Broadcast Channels,” *IEEE Trans. Info. Theory*, vol. 18, no. 1, pp. 2–14, 1972

[2] L. Li and A. Goldsmith, “Capacity and optimal resource allocation for fading broadcast channels. I. ergodic capacity,” *IEEE Trans. Info. Theory*, vol. 47, no. 3, pp. 1083–1102, 2001

[3] H. Viswanathan, S. Venkatesa, and H. Huang, “Downlink capacity evaluation of cellular networks with known interference cancellation,” *IEEE Jour. Selec. Areas in Comm.*, vol. 21, no. 5, pp. 802–811, 2003

[4] R. Knopp and P. Humblet, “Information capacity and power control in single cell multiuser communications,” *in Proc. IEEE Inter. Conf. Comm.*, vol. 1, pp. 331-335, 1995

[5] G. Caire.and S. Shamai, “On the achievable throughput of a multiantenna Gaussian broadcast channel”, *Information Theory, IEEE Transactions on* Volume 49, Issue 7, July 2003 Page(s):1691 - 1706

[6] H. Weingarten, Y. Steinberg and S. Shamai,”The capacity region of the Gaussian MIMO broadcast channel” *Information Theory, 2004. ISIT 2004. Proceedings. International Symposium on* 27 June-2 July 2004 Page(s):174

[7] M.Costa, “Writing on dirty paper”,*IEEE Trans. Inform. Theory*, vol. IT-29, pp. 439-441, May 1983.

[8] N. Jindal, S. Vishwanath, and A. Goldsmith, “On the duality of Gaussian multiple-access and broadcast channels” *IEEE Trans. Info. Theory*, vol. 50, no. 5, pp. 768–783, 2004.

[9] D.N.C. Tse and S.V. Hanly, “Multiaccess fading channels. I. Polymatroid structure, optimal resource allocation and throughput capacities”, *Information Theory, IEEE Transactions on* Volume 44, Issue 7, Nov. 1998 Page(s):2796 - 2815

[10] M. Sharif and B. Hassibi, “On the capacity of MIMO broadcast channels with partial side information”, *Information Theory, IEEE Transactions on*, Volume 51, Issue 2, Feb. 2005 Page(s):506 - 522

[11] N. Jindal and A. Goldsmith, “Capacity and optimal power allocation for fading broadcast channels with minimum rates, *IEEE Trans. Info. Theory*, vol. 49, no. 11, pp. 2895–2909, 2003

[12] M. Sharif, A. Dana and B. Hassibi, “Differentiated rate scheduling for Gaussian broadcast channels”, *Proceedings of the 2005 IEEE International Symposium on Information Theory*

[13] M. Sharif and B. Hassibi, ”Scaling laws of sum rate using time-sharing, DPC, and beamforming for MIMO broadcast channels”, *Information Theory, 2004. ISIT 2004. Proceedings. International Symposium on*, 27 June-2 July 2004 Page(s):175

[14] A. Lapidoth, S. Shamai and M.A. Wigger, “On the capacity of fading MIMO broadcast channels with imperfect transmitter side-information”, *Proceedings of the 43rd Allerton Conference on Communication, Control and Computing*, September 2005

[15] P. Viswanath, D.N.C. Tse and R. Laroia, “Opportunistic beamforming using dumb antennas”, *Information Theory, IEEE Transactions on* Volume 48, Issue 6, June 2002 Page(s):1277 - 1294

[16] A. Vakili, A. Dana, M. Sharif and B. Hassibi, “Differentiated rate scheduling for MIMO Gaussian broadcast channels”, *in preparation*.